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# The Role of the Signal–Noise Ratio in Cointegrated Systems*

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**Abstract**

This paper studies the impact of the signal–noise ratio on the estimates of the parameters of cointegrated systems in small samples. The concept of canonical correlations as exploited by the Johansen Maximum Likelihood procedure is proven to measure the signal–noise ratio. A Monte–Carlo study for cointegrated systems with small signal–noise ratios shows that the Johansen estimator has low biases although the Johansen tests underestimate the rank of the cointegrating space. In contrast, the Fully Modified OLS estimator is found to be significantly biased in certain cases. Re–analyzing and re–estimating with the FM–OLS estimator of two empirical studies illustrates these results.

KEYWORDS: Cointegration, Monte–Carlo Analysis, Fully Modified Estimation, Canonical Correlation, Small Sample Behavior.

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# 1 Introduction

Following the paper by Engle and Granger (1987), there have been many contributions to the statistical theory for the analysis of cointegrated systems, see inter alia Johansen (1988), Phillips (1991a) and the review in Banerjee, Dolado, Galbraith and Hendry (1993). In addition, numerous empirical studies have employed these techniques, e.g. Baba, Hendry and Starr (1992), Bårdsen and Fisher (1993), Boswijk (1992), Clements and Mizon (1990), Hansen (1992a,b), Hendry and Mizon (1993), Johansen and Juselius (1990), Kunst and Neusser (1990), Mizon (1991), Naug and Nymoen (1993) and Nymoen (1992). Most of these studies used the VAR representation for a cointegrated system involving $I(1)$ variables and adopted the Maximum Likelihood procedure developed by Johansen (1988). Fewer articles have applied the Fully Modified OLS estimator to the estimation of the long run parameters, which was proposed by Hansen (1992a), Phillips (1991a), and Phillips and Hansen (1990). Although the limiting distributions of these alternative estimators are well known, their finite sample behavior is less well analyzed.

Since applied work is always limited by the number of observations, there is the question as to whether the available information is sufficient to yield precise estimates of cointegrating vectors with small adjustment weights. Further, since the cointegrating vectors are not uniquely defined, the influence of a re–arrangement of the cointegrating vectors on their adjustment weights, as well as the dependence on the adjustment weights of the other cointegrating vectors, has to be analyzed. This paper deals with these issues by introducing the signal–noise ratio as a measure for the expected quality of the estimates of a cointegrated system. The signal–noise ratio is shown to be related to the eigenvalues of the Johansen test statistics. Attention is also focused on situations in which the rank of the cointegrating space is over–estimated.

The remainder of the paper is organized as follows: In section 2, the problem is treated theoretically. Section 3 contains a brief description of the Fully Modified OLS estimator and a discussion of its applicability. The behavior of the Johansen and the Fully Modified OLS estimator in systems with a low signal–noise ratio is illustrated by Monte–Carlo experiments reported in section 4. In section 5, the derived results are applied to two data sets as estimated in Clements and Mizon (1991) and Hendry and Mizon (1993). Section 6 concludes.

# 2 Theoretical approach to the signal–noise ratio

Without loss of generality for the following argument, the simple $N$ dimensional DGP

$$\Delta X_t = \alpha\beta' X_{t-1} + \epsilon_t \tag{1}$$

is considered. In equation (1), $\epsilon_t$ is $iid\, N(0, \Omega)$ and the adjustment weights $\alpha$ and the cointegrating vectors $\beta$ are $N \times r$ matrices.[1] Let $S_{ij}$ denote the product moment matrices

$$S_{ij} := T^{-1} \sum_{t=1}^{T} Z_{it} Z'_{jt}, \quad i, j \in \{0, 1\}, \tag{2}$$

where $Z_{0t} := \Delta X_t$ and $Z_{1t} := X_{t-1}$.

In order to understand the impact of the signal–noise ratio of the cointegrated system on its estimates, the limit product moment matrix

[1] A more general VAR representation is given by

$$\Delta X_t = \alpha\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \epsilon_t,$$

where $\epsilon_t$ is $iid\, N(0, \Omega)$ and the $d$ components in $D_t$ are a constant or centered deterministic terms like a linear trend or seasonal dummies. The regression of $\Delta X_t$ and $X_{t-1}$ on $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}$ and $D_t$ yields the residuals $R_{0t}$ and $R_{1t}$. With these definitions, the regression equation can be reformulated to

$$R_{0t} = \alpha\beta' R_{1t} + \hat{\epsilon_t}$$

for some Gaussian error process $\hat{\epsilon_t}$. If $\Gamma_i$ and $\Phi$ are unrestricted, this multivariate regression is for the estimation of $\alpha$ and $\beta$ equivalent to the DGP in (1).

$\Sigma_{00}$ of $S_{00}$ is transformed as follows:

$$\begin{aligned}\Sigma_{00} &:= \operatorname{p}\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \Delta X_t \Delta X_t' & (3)\\ &= \operatorname{p}\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} (\alpha\beta' X_{t-1} + \epsilon_t)(\alpha\beta' X_{t-1} + \epsilon_t)' & (4)\\ &= \operatorname{p}\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \alpha\beta' X_{t-1} X_{t-1}' \beta\alpha' + \epsilon_t\epsilon_t' & (5)\\ &=: \alpha\Sigma_{\beta'1\beta}\alpha' + \Omega, & (6)\end{aligned}$$

where $\Sigma_{\beta'1\beta} := \operatorname{p}\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} \beta' X_{t-1} X_{t-1}'\beta$. The definition $\Sigma_{\beta'1\beta}$ is employed since the limit of the product moment matrix of the non–stationary variable $X_{t-1}$ does not necessarily exist. The above equations give a decomposition of the covariance matrix of $\Delta X_t$ into a signal covariance matrix $\alpha\Sigma_{\beta'1\beta}\alpha'$ and an error covariance matrix $\Omega$. They imply that as long as the signal–component $\alpha\Sigma_{\beta'1\beta'}\alpha'$ dominates the noise of the error process $\Omega$, the estimates of the cointegrating vectors and adjustment coefficients are expected to be relatively precise even for small sample sizes.

In small samples, the problem of imprecise estimates for the cointegrating space arises if the signal–noise ratio matrix $SNR := (\Omega^{-1}\alpha\Sigma_{\beta'1\beta}\alpha' + I_N)$ is not significantly different from the identity matrix $I_N$. Using the eigenvalues as a matrix norm, an equivalent statement is:

$$\exists \mu_i \in \{\mu_j | \mu_j \text{ eigenvalue of } SNR \quad \text{such that } \mu_j \neq 1\}^2 \text{ and } \mu_i \not\gg 1. \quad (7)$$

For fixed and invertible $\Omega$ and normed $\Sigma_{\beta'1\beta} = I_r$, statement (7) means that the matrix of adjustment weights $\alpha$ has to have a rank lower than $r$ in the sense that the $r$th largest eigenvalue is small. Put differently, if, after normalization of the cointegrating vectors, the matrix of adjustment coefficients $\alpha$ of a DGP has rank less than $r$, then the signal–component of the cointegrating vectors will not easily be distinguishable from the noise of the error process. Consequently, the information from a small

[2]Note that the eigenvalues of the matrix $SNR$ have to be greater than or equal to one since the matrix $\Omega^{-1}\alpha\Sigma_{\beta'1\beta'}\alpha'$ is positive definite.

set of observations may not be sufficient to yield precise estimates of the cointegrating space.

The discussion of the signal–noise ratio of a cointegrated system can be embedded in the Johansen Maximum Likelihood approach for the estimation of the cointegrating vectors and adjustment coefficients. Johansen (1988) shows that under the hypothesis that $\alpha$ and $\beta$ are of rank $r$ the maximization of the likelihood function of equation (1) is equivalent to the minimization of

$$\frac{|S_{00}||\beta'(S_{11} - S_{10}S_{00}^{-1}S_{01})\beta|}{|\beta' S_{11}\beta|}. \tag{8}$$

Exploiting Anderson's (1958) idea of reduced rank regression, the solution of the minimization problem is given by

$$\hat{\beta} = (v_1, \ldots, v_r), \tag{9}$$

where $V = (v_1, \ldots, v_N)$ are the eigenvectors of the equation

$$|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}| = 0, \tag{10}$$

normed by $V'S_{11}V = I_N$ and ordered by $\lambda_1 > \ldots > \lambda_N > 0$. The adjustment coefficients result from

$$\hat{\alpha} = S_{01}\hat{\beta}(\hat{\beta}'S_{11}\hat{\beta})^{-1}. \tag{11}$$

If the long–run matrix $\alpha'\beta$ of the underlying DGP has rank $r$ the characteristic equation (10) will have $r$ eigenvalues different from zero.

In the theory of reduced rank regression, the eigenvalues $\lambda_i$ of equation (10) are also called the canonical correlations, because

$$\begin{aligned}
\lambda_i &= Corr(\alpha_i' S_{00}^{-1}\Delta X_t, \beta_i' X_{t-1}) && (12)\\
&= \alpha_i' S_{00}^{-1} S_{01}\beta_i && (13)\\
&= \alpha_i' S_{00}^{-1}\alpha_i. && (14)
\end{aligned}$$

The technique of reduced rank regression chooses successively the adjustment coefficients $\alpha_i$ and cointegrating vectors $\beta_i$ such that the linear combinations $\alpha_i' S_{00}^{-1}\Delta X_t$ and $\beta_i' X_{t-1}$ lead to a maximal correlation between

$S_{00}^{-1}\Delta X_t$ and $X_{t-1}$ and are uncorrelated with $\alpha'_j S_{00}^{-1}\Delta X_t$ and $\beta'_j X_{t-1}$ for $j = 1, \ldots, N; j \neq i$. In other words, if the $i$th eigenvalue of the characteristic equation (10), $\lambda_i$, is comparatively small, then there just exists a small correlation between the nonstationary variable $X_{t-1}$ multiplied by the cointegrating relationships ($\beta_i X_{t-1}$) and the orthogonal projection of $X_{t-1}$ onto the space spanned by the stationary variables $\Delta X_t$ multiplied by the cointegrating relationships ($\alpha'_i S_{00}^{-1}\Delta X_t = \beta_i S_{10} S_{00}^{-1}\Delta X_t$).

The concept of the canonical correlations resulting from reduced rank regression can be linked to the introduced measure of the signal–noise ratio of a cointegrated system. In order to formulate this connection, the canonical correlations $\lambda_i$ of the limit characteristic equation,[3]

$$\mathrm{p}\lim_{T\to\infty} |\lambda I_N - S_{01} S_{11}^{-1} S_{10} S_{00}^{-1}| = 0, \tag{15}$$

and the eigenvalues $\mu_i$ of the signal–noise ratio matrix $SNR$,

$$\left|\mu I_N - \left(\Omega^{-1}\alpha\Sigma_{\beta' 1 \beta}\alpha' + I_N\right)\right|, \tag{16}$$

are considered. Between these measures, the following equivalence can be established:

$$\lambda_i \text{ canonical correlation} \iff \mu_i = \frac{1}{1-\lambda_i} \text{ is eigenvalue of } SNR. \tag{17}$$

The $r$th largest eigenvalue $\lambda_r$ of the characteristic equation (15) is close to zero if and only if there exists an eigenvalue $\mu_r$ of the signal–noise ratio matrix $SNR$ (resulting from the characteristic equation (16)) which is close to one.

For the proof of (17), the cointegrating vectors are normed so that $\Sigma_{\beta' 1 \beta} = I_r$. Let $\lambda_i$ be an eigenvalue of the characteristic equation (15),

[3] The probability limits of $S_{11}$ and $S_{10}$ do not necessarily have to exist. Nevertheless, the eigenvalue problem of equation (10) can equivalently be transformed:

$$\begin{aligned} \iff \quad |\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| &= 0 \\ \iff \quad |\lambda I_N - S_{11}^{-1} S_{10} S_{00}^{-1} S_{01}| &= 0 \\ \iff \quad |\lambda I_N - S_{01} S_{11}^{-1} S_{10} S_{00}^{-1}| &= 0. \end{aligned}$$

The probability limit of the product matrix $S_{01} S_{11}^{-1} S_{10}$ is then well defined.

which can be interpreted as an asymptotic canonical correlation $\lambda_i = \alpha_i' \Sigma_{00}^{-1} \alpha_i$. The following equations are then equivalent:

$$\begin{aligned}
\Longleftrightarrow \quad & \left| \frac{1}{(1-\lambda_i)} I_N - \left(\Omega^{-1}\alpha\alpha' + I_N\right)^{-1} \right| &=& \ 0 \\
\Longleftrightarrow \quad & |(1-\lambda_i)I_N - (\Omega^{-1}\alpha\alpha' + I_N)| &=& \ 0 \\
\Longleftrightarrow \quad & \left| -\lambda_i I_N - \left((\Omega^{-1}\alpha\alpha' + I_N)^{-1} - I_N\right) \right| &=& \ 0 \\
\Longleftrightarrow \quad & \left| \lambda_i I_N - \underbrace{\Omega^{-1}\alpha\alpha'\left(\Omega^{-1}\alpha\alpha' + I_N\right)^{-1}}_{=:M} \right| &=& \ 0.
\end{aligned} \tag{18}$$

The proof of (17) is thus completed if $\lambda_i$ is shown to be an eigenvalue of the matrix $M$. The eigenvalue property of $\lambda_i$ for $M$ follows after multiplying $M$ by $\Omega^{-1}\alpha_i$ from the right:

$$M\Omega^{-1}\alpha_i = \Omega^{-1}\alpha\alpha'\left(\Omega^{-1}\alpha\alpha' + I_N\right)^{-1}\Omega^{-1}\alpha_i \tag{19}$$

$$= \Omega^{-1}\alpha\alpha'(\alpha\alpha' + \Omega)^{-1}\Omega\Omega^{-1}\alpha_i \tag{20}$$

$$= \lambda_i\Omega^{-1}\alpha_i. \tag{21}$$

Note that the last implication stems from:

$$\mathrm{p}\lim_{T\to\infty} \alpha'\Sigma_{00}^{-1}\left(T^{-1}\sum_{t=1}^{\infty}\Delta X_t \Delta X_t'\right)\Sigma_{00}^{-1\prime}\alpha = \alpha'(\alpha\Sigma_{\beta'1\beta}\alpha' + \Omega)^{-1}\alpha = \Lambda, \tag{22}$$

where $\Lambda$ is the diagonal matrix containing the eigenvalues $\lambda_i$ of the limit characteristic equation (15).[4]

The equivalence (17) relates the eigenvalues of the characteristic equation (15) directly to the eigenvalues of the signal–noise ratio matrix. It therefore allows to interpret them as measures of the signal–noise ratio of a cointegrated system. This interpretation can be transferred to the Johansen tests. The maximal eigenvalue test (for the null hypothesis of the existence of $r$ cointegrating vectors versus the hypothesis of the existence of $(r+1)$ cointegrating vectors) and the trace eigenvalue test (for the null hypothesis of the cointegrating space having rank $r$ versus

[4]Compare equation (14) expressing the eigenvalues $\lambda_i$ as the canonical correlations.

the unrestricted model) are given by

$$T\,ln\left(\frac{1}{(1-\lambda_{r+1})}\right) \quad \text{and} \quad T\sum_{i=r+1}^{N} ln\left(\frac{1}{(1-\lambda_i)}\right). \tag{23}$$

Both test statistics essentially depend on the number of the observations $T$ and the logarithms of the estimated eigenvalues of the SNR matrix $\frac{1}{(1-\lambda_i)}$. In order to exceed a critical value, either the number of observations has to be high or the logarithmed estimates of the eigenvalues have to be significantly different from one.

For cointegrated systems with high signal–noise ratios, the Johansen tests are therefore likely to spot the existence of the cointegrating relationships even for a small number of observations. In contrast, if the eigenvalues of the signal–noise ratio matrix $SNR$ are close to one, the cointegrating space may be underestimated by the Johansen test when the sample is small. Note that this effect will be intensified by the use of the adjusted statistics proposed by Reinsel and Ahn (1988).[5]

The preceding analysis has been concerned with the population characteristics. Hence it is relevant for all estimation methods. For a system with a low signal–noise ratio both Maximum Likelihood estimators (the Johansen estimator and the Fully Modified OLS estimator) are therefore likely to generate unreliable estimates of the cointegration space. Nevertheless, as will be seen in section 4, the Johansen estimator proves to be a very stable estimator even for systems with small signal–noise ratios, whereas the Fully Modified OLS estimator displays serious biases.

In case of an overestimated rank of the cointegrating space, the estimate of the $(r+1)$th cointegrating vector results from a mis–interpretation of the noise of the error process as a signal of the cointegrating space and thus cannot contain structural elements of the DGP. An important feature of the Johansen procedure is that the estimates of the first $r$ cointegrating vectors as eigenvectors matching the canonical correlations are not affected by an overestimation of the cointegrating space. This is not the case for the Fully Modified OLS estimator, which may give distorted estimates of the whole cointegrating space when its rank is overestimated.

[5] For the adjusted statistics the logarithm of the eigenvalues is multiplied by $(T - Np)$ where $p$ denotes the number of lags in the VAR.

This result can be confirmed by recalling the asymptotical distribution of the estimated cointegrating vectors. According to Johansen (1993), it is given by

$$\begin{bmatrix} T\gamma'(\hat{\beta}-\beta) \\ T^{3/2}\tau'(\hat{\beta}-\beta) \end{bmatrix} \xrightarrow{w} (I-\beta c')\left(\int_0^1 GG'dt\right)^{-1}\int_0^1 G(dV_\alpha), \tag{24}$$

where $\hat{\beta}$ and $\beta$ are normalized by $c'\beta = c'\hat{\beta} = I$ and $\gamma$ and $\tau$ are $N \times (N-r-1)$ and $N\times 1$ matrices resulting from the modified Engle–Granger Representation [Johansen (1991)]. $G := (G_1, G_2)$ is defined as

$$G_1(t) := \gamma' C(W(t) - \bar{W}), \tag{25}$$

$$G_2(t) := t - \frac{1}{2}, \tag{26}$$

for $W$ being a Brownian motion in $N$ dimensions on the unit interval, $C$ a matrix defined by the Engle–Granger Representation Theorem, and

$$V_\alpha = \left(\alpha'\Omega^{-1}\alpha\right)^{-1}\alpha'\Omega^{-1}W. \tag{27}$$

Thus, the distribution is mixed Gaussian with asymptotic quadratic variation

$$(I-\beta c')\left(\int_0^1 GG'dt\right)^{-1}(I-\beta c')\otimes\left(\alpha'\Omega^{-1}\alpha\right)^{-1}, \tag{28}$$

which can consistently be estimated by

$$\left(I-\hat{\beta}c'\right)S_{11}^{-1}\left(I-\hat{\beta}c'\right)\otimes\left(\hat{\alpha}'\hat{\Omega}^{-1}\hat{\alpha}\right)^{-1}. \tag{29}$$

For testing purposes, the distribution of $\hat{\beta}$ cannot be applied, because it might be infinite. For this reason, the values of $\beta$ have to be interpreted with a "variance" given in (29). Since the Johansen estimates of the adjustment coefficients are normally distributed [Johansen (1993)],

$$T^{\frac{1}{2}}\left(\hat{\alpha}-\alpha\right) \xrightarrow{w} N_{N\times r}\left(0, \Omega\otimes\Sigma_{\beta'1\beta}^{-1}\right), \tag{30}$$

the estimate of the $(r+1)$th adjustment coefficient is very likely to be close to zero. Therefore, the matrix $\hat{\alpha}'\hat{\Omega}^{-1}\hat{\alpha}$ may have one very small eigenvalue when the rank of the cointegrating space is overestimated. Consequently, the "variance" of $\hat{\beta}$, goes to infinity.

# 3 The Fully Modified OLS estimator

In order to summarize the estimation method of Phillips (1991a) and Phillips and Hansen (1990), it is convenient to adopt the notation of Phillips (1991a). They suggest a triangular system representation of a $N$ dimensional system of $I(1)$ variables with $r$ cointegrating relationships:

$$\Delta X_t \quad = \quad -EAX_{t-1} \quad + \quad v_t, \tag{31}$$

where

$$E := \begin{pmatrix} I_r \\ 0 \end{pmatrix} \quad \text{and} \quad A := \begin{pmatrix} I_r & -B \end{pmatrix}. \tag{32}$$

Phillips starts out with the prototypical case where the error process $v_t$ is $iid\, N(0, \Omega)$. He shows that the Maximum Likelihood estimator for $B$ is identical to the Ordinary Least Squares estimator of the linear system

$$X_{1t} = BX_{2t-1} + C\Delta X_{2t} + \tilde{v}, \tag{33}$$

where $C := \Omega_{12}\Omega_{22}^{-1}$ and $\tilde{v} := v_{1t} - \Omega_{12}\Omega_{22}^{-1}v_{2t}$. Consequently, an explicit formula for the Maximum Likelihood estimator is available in the prototypical case.

The triangular system representation for the case in which $v_t$ is stationary, but no longer $iid\, N(0, \Omega)$, can be estimated parametrically [Phillips (1991a)] or by a semiparametric correction [Phillips and Hansen (1990)]. To set up a parametric likelihood, it is assumed that the error process is generated by a parametric linear process

$$v_t = \sum_{j=0}^{\infty} D_j(\theta)\nu_{t-j}, \tag{34}$$

where $\nu_t$ is $iid(0, \Sigma_\nu(\theta))$, $\Sigma_\nu(\theta) > 0$, $D_0 = I$ and the coefficient matrices $D_j(\theta)$ satisfy

$$\sum_{j=0}^{\infty} j^{1/2}\|D_j(\theta)\| < \infty. \tag{35}$$

Following the approach of Dunsmuir and Hannan (1976) and Dunsmuir (1979), a good approximation for the Gaussian Likelihood is given by the

Whittle Likelihood.[6] Maximizing the Whittle Likelihood leads, as in the prototypical case, to an asymptotically efficient estimator of $B$, but does not yield an explicit expression for the estimator anymore.

The semiparametric estimator proposed by Phillips and Hansen (1990) is the Fully Modified OLS estimator. Phillips and Hansen assume that the innovation error process is strictly stationary and ergodic with zero mean, finite covariance matrix $\Omega$, and continuous spectral density matrix $f_{vv}(\lambda)$ with $\Gamma = 2\pi f_{vv}(0)$. Furthermore, the partial sum process constructed from the error process is supposed to satisfy the multivariate invariance principle

$$T^{-1/2}\sum_{1}^{[Tr]} v_j \qquad \Longrightarrow \qquad B(r) \equiv BM(\Gamma), \qquad 0 < r \leq 1. \qquad (36)$$

The long-run covariance matrix $\Gamma$ of the error process is then decomposed into

$$\Gamma = \Omega + \Lambda + \Lambda' \qquad (37)$$

$$:= E(v_1 v_1') + \sum_{k=2}^{\infty} E(v_1 v_k') + \sum_{k=2}^{\infty} E(v_k v_1'). \qquad (38)$$

[6]The Whittle Likelihood is given by

$$L_T(B,\theta) = -ln\,|\Sigma_\nu(\theta)| - T^{-1}\sum_{s} tr\left\{f(\lambda_s;\theta)^{-1} I(\lambda_s)\right\}, \qquad -T/2 < s \leq [T/2].$$

In the expression above the spectral density matrix of $v_t$ is

$$f(\lambda;\theta) = (1/2\pi)D(e^{i\lambda};\theta)\Sigma_\nu(\theta)D(e^{i\lambda};\theta)^{\star}, \qquad D(z;\theta) = \sum_{0}^{\infty} D_j(\theta)z^j,$$

where "$\star$" denotes the complex conjugate transpose of a matrix and the periodogram at frequency $\lambda \in (-\pi,\pi]$ is

$$I(\lambda) = w(\lambda)w(\lambda)^{\star}.$$

Furthermore,

$$w(\lambda) = (2\pi T)^{-1/2}\sum_{1}^{T}(\Delta X_t + EAX_{t-1})e^{it\lambda}$$

is the density function and $\lambda_s = 2\pi s/T$ are the fundamental Fourier frequencies for $-T/2 < s \leq [T/2]$.

Phillips and Hansen use consistent estimates of $\Omega, \Lambda$ and $\Gamma$ for the construction of the Fully Modified OLS estimator:

$$\begin{aligned} \hat{B} &= \left[\sum_{1}^{T} X_{1t}^{+} X_{2t}' - T\left[I_r, -\hat{\Gamma}_{12}\hat{\Gamma}_{22}^{-1}\right]\left[\hat{\Omega}_{21} + \hat{\Lambda}_{21}, \hat{\Omega}_{22} + \hat{\Lambda}_{22}\right]'\right] \\ &\quad \left[\sum_{1}^{T} X_{2t}X_{2t}'\right]^{-1}. \end{aligned} \tag{39}$$

In equation (39), the superscript $\hat{}$ denotes the consistent estimates, $X_{1t}^{+} := X_{1t} - \hat{\Gamma}_{12}\hat{\Gamma}_{22}' \Delta X_{2t}$ and $\Gamma, \Lambda$ and the $\Omega$ matrices are partioned conformably with $X_t$.

Since in most of the applications, the cointegrating residuals $v_t$ have a significant degree of correlation, Hansen (1992a) suggests to prewhiten the residuals by a VAR(1):

$$\hat{v_t} = \Phi \hat{v_{t-1}} + \hat{\epsilon}_t \tag{40}$$

He then applies the kernel estimation to the whitened residuals $\hat{\epsilon}$. The covariance parameter of interest can be obtained by recoloring:

$$\hat{\Gamma} = (I - \hat{\Phi})^{-1}\hat{\Gamma}_{\epsilon}(I - \hat{\Phi}')^{-1} \tag{41}$$

and

$$\hat{\Omega} + \hat{\Lambda} = (I - \hat{\Phi}')^{-1}(\hat{\Omega}_{\epsilon} + \hat{\Lambda}_{\epsilon}) - (I - \hat{\Phi})^{-1}\hat{\Phi}T^{-1}\sum_{i=1}^{T}\hat{v}_t\hat{v}_t{}'. \tag{42}$$

In Kostial and Mizon (1993), it is shown that for any $N$ dimensional $I(1)$ system of variables with $r$ cointegrating relationships there exists a permutation of the variables such that the system can be written in the triangular form (31) used by the Fully Modified OLS estimator. Nevertheless, if one of the variables is stationary, there exist permutations of the variables such that the system cannot be represented by equation (31). An estimation of this special ordering of the variables will give misleading results. Consider for example the following system in triangular form

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ \Delta X_{3t} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha X_{3t} \\ 0 \end{pmatrix} + \epsilon_t. \tag{43}$$

It generates one stationary variable, $X_{1t}$, and two cointegrated $I(1)$ variables, $X_{2t}$ and $X_{3t}$. If the stationarity of $X_{1t}$ is not known and if $X_{1t}$ enters the system as the driving variable, then the Fully Modified OLS estimator will produce completely distorted results.

In order to avoid this problem with the triangular system representation, the Fully Modified OLS estimator should not just be applied to the original data set but to transformations of the data set achieved by application of permutation matrices. Estimation results derived by the Fully Modified OLS estimator should only be accepted, if they are invariant under different permutations or — in the case of differing outcomes — are explainable as a result of the mis–specification mentioned above.

Moreover, if the rank of the cointegrating space is underestimated, it is possible that the estimation of the differently permutated data leads to estimates of different vectors in the cointegrating space. Therefore, the estimation of different permutations of the data set can help to determine the rank of the cointegrating space.

This is illustrated in Kostial (1993) through a re–estimation of the data set used in Clements and Mizon (1991) and Hendry and Mizon (1993) with the semi–parametric Fully Modified OLS estimator. The non–parametric estimates of the long–run variance–covariance matrix (37) needed for the Fully Modified OLS estimator are derived by kernel estimation using an automatic choice of the bandwidth parameters as suggested by Andrews (1991). The estimation was done for different kernels (Bartlett, Parzen and Quadratic Spectral kernel), for different assumptions on the rank of the cointegrating space and for all possible permutations of subsets of driving variable and non–driving variables.[7]

# 4 Monte–Carlo experiments

After the theoretical treatment of the signal–noise ratio of a cointegrated system and its relation to the canonical correlations of the Johansen es-

[7]Kostial (1993) also includes a summary of Monte–Carlo experiments on the behavior of the Fully Modified OLS estimator.

timation procedure, Monte–Carlo experiments are used to illustrate the problem and to analyze its relevance for applied work. Two different issues will be discussed, the first being the precision of the Johansen and the Fully Modified OLS estimates and the second being the correct rejection of the null hypotheses of the Johansen tests. The main questions to be answered by the Monte–Carlo experiments are:

- How precise are the estimates of the eigenvalues of the limit characteristic equation and thus the signal–noise ratio matrix?
- When is the signal–noise ratio large enough to ensure precise estimates via the Johansen Maximum Likelihood and the Fully Modified OLS estimator?
- Provided that the Johansen tests underestimate the correct rank of the cointegrating space, does the rejection by the Johansen tests coincide with imprecise estimates of the cointegrating vectors and adjustment coefficients?
- Does the precision of the Johansen estimate of a single cointegrating vector depend only on its canonical correlation or does it depend on the canonical correlations of the other cointegrating vectors, too?
- In the case of one relatively small eigenvalue of the limit characteristic equation: Are the estimates derived with the Fully Modified OLS estimator distorted when a lower rank of the cointegrating space than the rank of the cointegrating space of the underlying DGP is assumed?

### 1. Pilot study

As a pilot study, the following simple DGP is considered:

$$\Delta X_t = \epsilon_t; \tag{44}$$

it creates four not cointegrated $I(1)$ variables. As for any of the DGPs considered for the Monte–Carlo experiments, the error process $\epsilon_t$ is *iid* $N(0, I)$. A Monte–Carlo experiment with 10,000 replications for 100 observations was run.

Trivially, the signal–noise ratio matrix is the identity matrix and the canonical correlations of the limit characteristic equation are zero. Table 1 shows the estimates of the canonical correlations, the values of the Johansen tests and the rejection frequencies for the 5% significance level of the unadjusted and adjusted Johansen tests.[8] Although the canonical

Table 1: Johansen tests for the DGP equation (44)

| | | | | |
|---|---|---|---|---|
| can. corr. (MC st.error below): | 0.15 | 0.08 | 0.04 | 0.01 |
| | (0.00) | (0.00) | (0.00) | (0.00) |

| | maximal eigenvalue test | | | | trace eigenvalue test | | | |
|---|---|---|---|---|---|---|---|---|
| mean | 0.54 | 3.74 | 8.33 | 15.99 | 0.54 | 4.28 | 12.61 | 28.60 |
| st.err. | 0.01 | 0.02 | 0.03 | 0.05 | 0.01 | 0.02 | 0.04 | 0.07 |
| ua. r.f. | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| a. r.f. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

correlations and thus the eigenvalues of the signal–noise ratio matrix are significantly overestimated, the Johansen tests correctly do not reject the hypothesis of no cointegrating relationships between the variables.

**2. Four dimensional DGP with one cointegrating relationship**

In a next step, a four dimensional DGP with one cointegrating relationship occurring in one equation is studied:

$$\Delta X_t = \begin{pmatrix} -0.2 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix} \begin{pmatrix} 1.0 & -1.0 & 1.0 & 1.0 \end{pmatrix} X_{t-1} + \epsilon_t. \tag{45}$$

In particular, the Monte–Carlo experiment was conducted for 80 observations and 10500 replications.[9] For this DGP, the non–zero eigenvalue of the limit characteristic equation is 0.32, implying a non–unity eigenvalue of 1.47 for the signal–noise ratio matrix $SNR$. Using the eigenvalues

[8] Adjusted tests as proposed by Reinsel and Ahn (1988).

[9] Tables of the following Monte–Carlo experiments are provided in the appendix.

of the signal–noise ratio matrix for formulae (23), equating them with the 5% critical values of the Johansen tests, and solving for the number of observations $T$, a rejection of the null hypothesis of no cointegrating relationship by the maximal eigenvalue test statistic requires at least 70 observations. A rejection by the trace statistic even needs 124 observations. For the adjusted Johansen tests, slightly higher numbers of observations are required. Thus, for 80 observations it is expected that the Johansen maximal eigenvalue test rejects the null hypothesis for almost every replication and that the Johansen trace test has a very small rejection frequency.

The results reported in the table do not confirm the expectations formulated above because the estimates of the eigenvalues of the signal–noise ratio matrix $SNR$ seem to vary noticeable over the replications. On average they are significantly higher than expected. The rejection frequencies for the trace statistic are thus high.[10] Nevertheless, in about 40% of the replications the adjusted maximal eigenvalue test does not reject the null hypothesis of no cointegrating relationship, implying that the canonical correlations were underestimated for these cases. Comparing the Monte–Carlo mean of the non–zero eigenvalue of the signal–noise ratio matrix with the Monte–Carlo mean of the maximal eigenvalue of the pilot study, it becomes rather obvious that even in the case of a non–rejection by the Johansen tests the estimated non–zero eigenvalue might be too high to be caused only by noise.

The Monte–Carlo mean of the Johansen estimate of the cointegrating vector is slightly biased with a small Monte–Carlo standard error. Even for a Monte–Carlo experiment with 70 observations (which is not reported in the appendix) the bias is less than 0.07.[11] Thus, although the null hypothesis of no cointegrating relationship may not be rejected by the Johansen tests, the Johansen estimate of the cointegrating vector is relatively precise. This is not the case for the estimates derived by the

[10] Note that the overestimation of the eigenvalues leads to an overestimation of the adjustment coefficients, too.

[11] Note that although there does not exist an asymptotic standard error of the Johansen estimates $\hat{\beta}$ (the asymptotic distribution is mixed Gaussian) the standard error of the Monte–Carlo experiment is well defined.

Fully Modified OLS estimator; these estimates display serious biases and high Monte–Carlo standard errors independently of the kernel used for the estimation of the long–run variance–covariance matrix.[12]

Note that for this experiment as well as for the following Monte–Carlo experiments, the standard error of the estimates of the adjustment coefficients is very small. This is due to the fact that the Johansen estimates of the adjustment coefficients are normally distributed as mentioned in section 2.

### 3. Four dimensional DGP with more than one cointegrating relationship

Before turning to the analysis of the results for higher dimensional cointegrating spaces, the problem of the unidentification of individual cointegrating relations derived by the Johansen procedure[13] needs to be discussed. In this paper the identification problem is treated differently for the two and the three dimensional cointegrating space. For the case of a two–dimensional cointegrating space, the following restrictions are imposed on the cointegrating space

$$\beta_{21} = 0 \qquad \beta_{12} = 0 \tag{46}$$

and the switching algorithm as proposed by Johansen (1992) is applied. For every Monte–Carlo replication, the same starting vector is used. This, of course, can lead to non–convergence.[14] To avoid these problems for the Monte–Carlo simulations, a new replication is started and the old discarded if the algorithm does not converge after 25 iterations.

For the identification of the three–dimensional cointegrating space, a proposal by Hargreaves (1994) is adopted. He exploits the fact that the cointegrating space as a whole is identified and a three–dimensional subspace (through the origin) in a four–dimensional space is uniquely

[12] It should be mentioned that there is no dynamics in the DGP. The inclusion of dynamics would lead to a worse performance of the Fully Modified OLS estimator [Inder (1993)].

[13] It should be noted that, unless the requirement of orthogonal eigenvectors is dropped, the eigenvectors are identified. For further details compare Phillips (1991b).

[14] The stopping criterion for the algorithm is that the absolute value of subsequent estimates is less than $10^{-2}$.

characterized by the vector which is perpendicular to it. Hence, the quality of estimation by the unidentified Johansen procedure can be measured by the angle between the perpendicular vector to the cointegrating space as given by the DGP and the perpendicular vector to the estimated cointegrating space.

The next Monte–Carlo experiments deal with a *four dimensional system with two cointegrating relationships.* The DGP is given by

$$\Delta X_t = \begin{pmatrix} -0.2 & 0.0 \\ -0.1 & -0.3 \\ 0.0 & 0.0 \\ 0.0 & 0.1 \end{pmatrix} \begin{pmatrix} 1.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 1.0 \end{pmatrix} X_{t-1} + \epsilon_t. \tag{47}$$

Monte–Carlo experiments were run for 60 and 100 observations. The asymptotic canonical correlations are 0.451 and 0.113; for these values a rejection of the maximal eigenvalue test of the hypothesis of the rank of the cointegrating space being less than two at the 5 % critical value needs 175 observations.

First consider the Monte–Carlo experiment with 60 observations. The tables in the appendix report the unrestricted Johansen estimates as well as the Johansen estimates under the restriction (46). As observed for the DGP (45), there occurs a significant overestimation of the canonical correlations. Nevertheless, the overestimation is not significant enough to lead to a rejection of the null hypothesis of the rank of the cointegrating space being less than two; just in 9 % of the replication the maximal eigenvalue test rejects correctly the null hypothesis. Note that even the null hypothesis of no cointegrating relationship is not rejected by both unadjusted tests for about 10 % of the replications indicating that for these replications the asymptotic canonical correlations are significantly underestimated.

Although for most of the replications, the Johansen tests are not able to spot the existence of the two–dimensional cointegrating space, the precision of the unrestricted as well as of the restricted Johansen estimates is rather high with a low Monte–Carlo standard error even for the small sample size of 60. Normalization of the unrestricted Johansen estimates leads to a certain bias. Therefore, it seems to be the case that the quality of estimation of a certain cointegrating vector is not just determined by its own canonical correlation, but also by the canonical correlations of the other cointegrating vectors. This observation confirms the analysis of Phillips (1991b). Increasing the number of observations from 60 to 100 does not lead to a significant improvement in the precision of the estimates and the Monte–Carlo standard errors but to a better performance of the Johansen tests.

For 60 observations the Fully Modified OLS estimator displays serious biases with high Monte–Carlo standard errors independent of the assumption on the rank of the cointegrating space. Thus, an underestimation of the rank of the cointegrating space does not necessarily involve an improvement in the precision of the estimates of the cointegrating vectors. This result leads to the conclusion that for every DGP with a small signal–noise ratio, the Fully Modified OLS estimator is not an appropriate estimator. Note that the biases of the Fully Modified OLS estimator are not unreasonable so that the Fully Modified OLS estimator can be used to confirm the results of the Johansen estimator qualitatively. The extension to 100 observations leads to a significant improvement of the biases but not to smaller Monte–Carlo standard errors.

Next consider a *four–dimensional DGP with three cointegrating relationships*

$$\Delta X_t = \begin{pmatrix} -0.2 & 0.0 & 0.0 \\ -0.1 & -0.3 & 0.0 \\ 0.0 & 0.0 & -0.25 \\ 0.0 & 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} 1.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 1.0 \end{pmatrix} X_{t-1} + \epsilon_t \quad (48)$$

for 60 observations. The table in the appendix provides the unrestricted Johansen estimates with the angle between the estimated and the true three–dimensional cointegrating space as described above. Although the system has two relatively small asymptotic canonical correlations (0.668, 0.155, 0.124) the precision of the normed estimate of the cointegrating vectors is very high with small Monte–Carlo variances.

This result is confirmed by the angle: 90 percent of the replications have an angle of less than 8 degrees and the mean angle is less than

4 degrees. Nevertheless, one has to be careful with the acceptance of the Johansen estimates since they display serious outliers as seen by the maximum angle. The outlier behavior is analytically explained by Phillips (1992). He shows that in the case of the data being generated by $N$ independent random walks, the distribution of the Johansen estimator is "matrix Cauchy" and has no finite first moments. The Cauchy–like tail behavior remains if the assumption of the $N$ independent random walks is relaxed.

Analogously to the Johansen estimator, the estimates derived by the Fully Modified OLS estimator display a relatively small bias but still with a high Monte–Carlo standard error. Therefore, a coincidence of the two estimators gives evidence that the estimated cointegrating space of the Johansen procedure is not just due to an outlier.

Two extra comments on the Monte–Carlo experiment for the three–dimensional cointegrating space should be appended. First, the performance of the Johansen tests is not convincing. They are not at all able to spot the existence of the three–dimensional cointegrating space and hardly able to spot the existence of a second cointegrating vector. Second, the Monte–Carlo experiment suggests the conclusion that the higher is the rank of the cointegrating space in relation to the dimension of the system, the higher the precision of the estimators.

# 5 Empirical illustration

For an empirical illustration Clements and Mizon (1991) and Hendry and Mizon (1993) are considered.[15] Clements and Mizon analyze the determination of earnings, prices, productivity and unemployment in the UK. The Johansen tests confirm the existence of at least three cointegrating vectors and the existence of a fourth cointegrating vector is rejected at the 5% significance level. The estimated canonical correlations are 0.58, 0.39, 0.20 and 0.12. The Monte–Carlo experiments of the previous section suggest that the rank of the cointegrating space might be four.

[15] Detailed results are contained in Kostial (1993).

Furthermore, if there exists a four–dimensional cointegrating space, it is expected from the Monte–Carlo experiments that the Fully Modified OLS estimator should produce the same results. In line with these results, the re–estimation of the data set by the Fully Modified OLS estimator shows clearly the existence of a four dimensional cointegrating space and confirms the estimates by the Johansen procedure.

Hendry and Mizon (1993) interrelate money, prices, income and interest rates in the UK. The Johansen tests for their data indicate one cointegrating vector. The second largest eigenvalue is significant at the 5 % level on the maximal eigenvalue test but the trace test rejects the hypothesis of a two–dimensional cointegrating space. The estimated canonical correlations are 0.42, 0.27, and 0.07. The Monte–Carlo experiments in the preceding section suggest that there may exist two cointegrating vectors and that it is unlikely that the Fully Modified OLS estimator is able to reproduce the results derived by the Johansen procedure. However, it should be able to confirm the assumption that the rank of the cointegrating space is two and to confirm the signs of the cointegrating vectors. Assuming rank one of the cointegrating space, the Fully Modified OLS estimator varies significantly between the different permutations and the different kernels used for the estimation of the long–run variance–covariance matrix of the error process. This is a clear sign for an underestimation of the rank of the cointegrating space. A re–estimation with the Fully Modified OLS estimator under the assumption of a two dimensional cointegrating space yields less, but still, varying estimates. The rank of the cointegrating space is therefore likely to be two. Finally, the signs of the cointegrating relationships estimated by the Johansen procedure are confirmed by the Fully Modified OLS estimator.

# 6 Conclusion

In a theoretical analysis of the role of the signal–noise ratio in the estimation of cointegrated systems, it has been shown that the canonical correlations as estimated by the Johansen procedure are perfect measures of the eigenvalues of the signal–noise ratio matrix. For systems with small eigenvalues of the signal–noise ratio matrix, Monte–Carlo experiments were run to illustrate the problem for the Johansen and the Fully Modified OLS estimator. Moreover, two empirical studies involving cointegrated systems were re–analyzed under the light of the signal–noise ratio. Four conclusions followed from the subsequent analysis of systems with small eigenvalues of the signal–noise ratio matrix:

(*i*) The Johansen tests tend to underestimate of the rank of the cointegrating space.

(*ii*) The higher is the rank of the cointegrating space $r$ in relation to the dimension of the system $N$, the higher the precision of the estimators.[16]

(*iii*) For small sample sizes, the Fully Modified OLS estimator shows significant biases, whereas the Johansen estimator displays — if at all — very small biases.

(*iv*) If the rank of the cointegrating space is underestimated, the biases of the Fully Modified OLS estimator remain.

Summarizing, the Johansen estimator appears to be superior to the Fully Modified OLS estimator for systems with small signal–noise ratios when the sample size is small. Moreover, since the Johansen test statistics tend to underestimate the rank of the cointegrating space, the rejection or acceptance of the Johansen estimates of the cointegrating vectors should additionally be confirmed by testing their stationarity.

[16] Intuitively, this result is not surprising since the system is getting closer to stationarity.

# Appendix

**Table 2: Monte–Carlo experiment for DGP equation (45) for 80 observations**

**Johansen tests**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| eigenvalues (MC st.err. below): | | | | | 0.37 (0.00) | 0.13 (0.00) | 0.05 (0.00) | 0.01 (0.00) |
| | maximal eigenvalue test | | | | trace eigenvalue test | | | |
| mean | 0.61 | 4.41 | 11.02 | 37.12 | 0.61 | 5.02 | 16.04 | 53.16 |
| st.err. | 0.01 | 0.02 | 0.04 | 0.09 | 0.01 | 0.02 | 0.05 | 0.11 |
| ua. rej.f. | 0.00 | 0.00 | 0.02 | 0.85 | 0.00 | 0.00 | 0.01 | 0.64 |
| a. rej.f. | 0.00 | 0.00 | 0.00 | 0.61 | 0.00 | 0.00 | 0.00 | 0.23 |

**Johansen estimates (MC stand. dev. below)**

$$\hat{\alpha} = \begin{pmatrix} -0.22 \\ (0.00) \\ 0.00 \\ (0.00) \\ -0.01 \\ (0.00) \\ -0.00 \\ (0.00) \end{pmatrix} \quad \hat{\beta} = \begin{pmatrix} 1.00 & -0.99 & 1.03 & 0.99 \\ (0.00) & (0.03) & (0.02) & (0.04) \end{pmatrix}$$

**Fully Modified OLS estimates (MC stand. dev. below)**

Bartlett kernel $$\hat{\beta} = \begin{pmatrix} 1.00 & -0.87 & 0.85 & 0.86 \\ (0.00) & (0.12) & (0.12) & (0.12) \end{pmatrix}$$

Parzen kernel $$\hat{\beta} = \begin{pmatrix} 1.00 & -0.87 & 0.85 & 0.86 \\ (0.00) & (0.12) & (0.12) & (0.12) \end{pmatrix}$$

Quadratic Spectral kernel $$\hat{\beta} = \begin{pmatrix} 1.00 & -0.87 & 0.85 & 0.86 \\ (0.00) & (0.12) & (0.12) & (0.12) \end{pmatrix}$$

**Table 3: Monte–Carlo experiment for DGP equation (47) for 60 observations**

**Johansen tests**

| | | | | |
|---|---|---|---|---|
| eigenvalues (MC st.err. below): | 0.50 | 0.21 | 0.09 | 0.01 |
| | (0.00) | (0.00) | (0.00) | (0.00) |

| | maximal eigenvalue test | | | | trace eigenvalue test | | | |
|---|---|---|---|---|---|---|---|---|
| mean | 0.71 | 5.42 | 14.67 | 43.02 | 0.71 | 6.13 | 20.80 | 63.81 |
| st.err. | 0.01 | 0.03 | 0.03 | 0.11 | 0.01 | 0.03 | 0.06 | 0.13 |
| ua. rej.f. | 0.00 | 0.00 | 0.09 | 0.95 | 0.00 | 0.00 | 0.05 | 0.89 |
| a. rej.f. | 0.00 | 0.00 | 0.01 | 0.70 | 0.00 | 0.00 | 0.00 | 0.04 |

**unrestricted Johansen estimates (MC st.err. below)**

$$\hat{\alpha} = \begin{pmatrix} -0.39 & 0.33 \\ (0.00) & (0.00) \\ -0.71 & -0.15 \\ (0.00) & (0.00) \\ 0.00 & 0.00 \\ (0.00) & (0.00) \\ 0.17 & 0.10 \\ (0.00) & (0.00) \end{pmatrix} \quad \hat{\beta} = \begin{pmatrix} 0.22 & 0.29 & 0.00 & 0.49 \\ (0.00) & (0.00) & (0.00) & (0.00) \\ -0.41 & 0.42 & -0.00 & 0.01 \\ (0.00) & (0.00) & (0.00) & (0.00) \end{pmatrix}$$

**normed unrestricted Johansen estimates (MC st.err. below)**

$$\hat{\alpha} = \begin{pmatrix} -0.24 & 0.02 \\ (0.00) & (0.00) \\ -0.09 & -0.34 \\ (0.00) & (0.00) \\ 0.00 & 0.00 \\ (0.00) & (0.00) \\ -0.01 & 0.00 \\ (0.00) & (0.01) \end{pmatrix} \quad \hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.00 & 1.08 \\ (0.00) & (0.00) & (0.05) & (0.05) \\ 0.00 & 1.00 & 0.01 & 0.96 \\ (0.00) & (0.00) & (0.03) & (0.03) \end{pmatrix}$$

**restricted Johansen estimates (MC st.err. below)**

$$\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & 0.001 & 0.98 \\ (0.00) & (0.00) & (0.01) & (0.01) \\ 0.00 & 1.00 & -0.00 & 1.01 \\ (0.00) & (0.00) & (0.00) & (0.00) \end{pmatrix}$$

**continuation of table 3: Monte–Carlo experiment for DGP equation (47) for 60 observations**

**FM–OLS estimates for one cointegrating relationship (MC stand. dev. below)**

Bartlett kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.30 & -0.02 & 0.53 \\ (0.00) & (0.06) & (0.05) & (0.09) \end{pmatrix}$

Parzen kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.29 & -0.02 & 0.54 \\ (0.00) & (0.06) & (0.05) & (0.09) \end{pmatrix}$

Quadratic Spectral kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.30 & -0.02 & 0.53 \\ (0.00) & (0.06) & (0.05) & (0.09) \end{pmatrix}$

**FM–OLS estimates for two cointegrating relationships (MC stand. dev. below)**

Bartlett kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.86 \\ (0.00) & (0.00) & (0.04) & (0.12) \\ 0.00 & 1.00 & 0.02 & 0.96 \\ (0.00) & (0.00) & (0.03) & (0.13) \end{pmatrix}$

Parzen kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.86 \\ (0.00) & (0.00) & (0.04) & (0.12) \\ 0.00 & 1.00 & 0.02 & 0.96 \\ (0.00) & (0.00) & (0.03) & (0.13) \end{pmatrix}$

Quadratic Spectral kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.86 \\ (0.00) & (0.00) & (0.04) & (0.12) \\ 0.00 & 1.00 & 0.02 & 0.96 \\ (0.00) & (0.00) & (0.03) & (0.13) \end{pmatrix}$

**Table 4: Monte–Carlo experiment for DGP equation (47) for 100 observations**

**Johansen tests**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| eigenvalues (MC st.err. below): | | | | | 0.48 (0.00) | 0.17 (0.00) | 0.05 (0.00) | 0.01 (0.00) |
| | maximal eigenvalue test | | | | trace eigenvalue test | | | |
| mean | 0.69 | 5.56 | 18.73 | 66.23 | 0.69 | 6.25 | 24.98 | 91.21 |
| st.err. | 0.01 | 0.03 | 0.05 | 0.14 | 0.01 | 0.03 | 0.06 | 0.16 |
| ua. rej.f. | 0.00 | 0.01 | 0.28 | 1.00 | 0.00 | 0.00 | 0.15 | 1.00 |
| a. rej.f. | 0.00 | 0.00 | 0.11 | 1.00 | 0.00 | 0.00 | 0.00 | 0.73 |

**unrestricted Johansen estimates (MC st.err. below)**

$$\hat{\alpha} = \begin{pmatrix} -0.40 & 0.34 \\ (0.00) & (0.00) \\ -0.78 & -0.15 \\ (0.00) & (0.00) \\ -0.00 & 0.00 \\ (0.00) & (0.00) \\ 0.18 & 0.11 \\ (0.00) & (0.00) \end{pmatrix} \qquad \hat{\beta} = \begin{pmatrix} 0.20 & 0.32 & 0.00 & 0.52 \\ (0.00) & (0.00) & (0.00) & (0.00) \\ -0.41 & 0.42 & -0.00 & 0.01 \\ (0.00) & (0.00) & (0.00) & (0.00) \end{pmatrix}$$

**normed unrestricted Johansen estimates (MC st.err. below)**

$$\hat{\alpha} = \begin{pmatrix} -0.23 & 0.02 \\ (0.00) & (0.00) \\ -0.10 & -0.33 \\ (0.00) & (0.00) \\ -0.00 & 0.00 \\ (0.00) & (0.00) \\ -0.01 & 0.10 \\ (0.00) & (0.01) \end{pmatrix} \qquad \hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.02 & 1.00 \\ (0.00) & (0.00) & (0.10) & (0.04) \\ 0.00 & 1.00 & 0.04 & 1.01 \\ (0.00) & (0.00) & (0.06) & (0.02) \end{pmatrix}$$

**restricted Johansen estimates (MC st.err. below)**

$$\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & 0.001 & 1.01 \\ (0.00) & (0.00) & (0.00) & (0.00) \\ 0.00 & 1.00 & -0.00 & 1.00 \\ (0.00) & (0.00) & (0.00) & (0.00) \end{pmatrix}$$

**continuation of table 4: Monte–Carlo experiment for DGP equation (47) for 100 observations**

**FM–OLS estimates for one cointegrating relationship (MC stand. dev. below)**

Bartlett kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.27 & -0.02 & 0.66 \\ (0.00) & (0.06) & (0.03) & (0.10) \end{pmatrix}$

Parzen kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.26 & -0.01 & 0.67 \\ (0.00) & (0.06) & (0.03) & (0.10) \end{pmatrix}$

Quadratic Spectral kernel $\hat{\beta} = \begin{pmatrix} 1.00 & -0.27 & -0.01 & 0.66 \\ (0.00) & (0.06) & (0.03) & (0.10) \end{pmatrix}$

**FM—OLS estimates for two cointegrating relationships (MC stand. dev. below)**

Bartlett kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.94 \\ (0.00) & (0.00) & (0.03) & (0.12) \\ 0.00 & 1.00 & 0.00 & 0.99 \\ (0.00) & (0.00) & (0.02) & (0.13) \end{pmatrix}$

Parzen kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.94 \\ (0.00) & (0.00) & (0.03) & (0.12) \\ 0.00 & 1.00 & 0.00 & 0.99 \\ (0.00) & (0.00) & (0.02) & (0.13) \end{pmatrix}$

Quadratic Spectral kernel $\hat{\beta} = \begin{pmatrix} 1.00 & 0.00 & -0.01 & 0.94 \\ (0.00) & (0.00) & (0.03) & (0.12) \\ 0.00 & 1.00 & 0.00 & 0.99 \\ (0.00) & (0.00) & (0.02) & (0.13) \end{pmatrix}$

**Table 5: Monte–Carlo experiment for DGP equation (48) for 60 observations**

**Johansen tests**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| eigenvalues (MC st.err. below): | | | | | 0.67 (0.00) | 0.26 (0.00) | 0.15 (0.00) | 0.02 (0.00) |
| | maximal eigenvalue test | | | | trace eigenvalue test | | | |
| mean | 1.15 | 9.99 | 18.59 | 69.90 | 1.15 | 11.14 | 29.74 | 99.64 |
| st.err. | 0.01 | 0.03 | 0.05 | 0.20 | 0.01 | 0.03 | 0.07 | 0.22 |
| ua. rej.f. | 0.00 | 0.08 | 0.27 | 1.00 | 0.00 | 0.04 | 0.38 | 1.00 |
| a. rej.f. | 0.00 | 0.01 | 0.04 | 0.97 | 0.00 | 0.00 | 0.00 | 0.57 |

**Johansen estimates (MC stand. dev. below)**

$$\hat{\alpha} = \begin{bmatrix} 0.07 & -0.07 & -0.05 \\ (0.00) & (0.00) & (0.00) \\ 0.10 & 0.13 & -0.04 \\ (0.00) & (0.00) & (0.00) \\ 0.08 & -0.11 & 0.08 \\ (0.00) & (0.00) & (0.00) \\ -0.05 & -0.01 & -0.02 \\ (0.21) & (0.00) & (0.01) \end{bmatrix} \quad \hat{\beta} = \begin{bmatrix} 1.00 & 0.02 & -1.31 & 18.22 \\ (0.00) & (0.38) & (0.44) & (17.41) \\ 8.18 & 1.00 & 0.42 & -9.79 \\ (4.77) & (0.00) & (0.66) & (9.24) \\ 7.48 & -0.68 & 1.00 & 8.26 \\ (5.21) & (0.31) & (0.00) & (7.64) \end{bmatrix}$$

**normed Johansen estimates (MC st.err. below)**

$$\hat{\alpha} = \begin{bmatrix} -0.26 & 0.02 & 0.02 \\ (0.00) & (0.00) & (0.00) \\ -0.10 & -0.36 & 0.02 \\ (0.00) & (0.00) & (0.00) \\ 0.02 & 0.02 & -0.31 \\ (0.00) & (0.00) & (0.00) \\ -0.02 & 0.09 & 0.10 \\ (0.00) & (0.00) & (0.00) \end{bmatrix} \quad \hat{\beta} = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.98 \\ (0.00) & (0.00) & (0.00) & (0.01) \\ 0.00 & 1.00 & 0.00 & 1.00 \\ (0.00) & (0.00) & (0.00) & (0.01) \\ 0.00 & 0.00 & 1.00 & 0.99 \\ (0.00) & (0.00) & (0.00) & (0.01) \end{bmatrix}$$

**Angle betw. est. and true cointegrating space in radian**

| mean | st.err. | min. | max. | $q05$ | $q10$ | $q50$ | $q90$ | $q95$ |
|---|---|---|---|---|---|---|---|---|
| 0.068 | 0.001 | 0.001 | 1.533 | 0.185 | 0.132 | 0.044 | 0.016 | 0.012 |

**continuation of table 5: Monte–Carlo experiment for DGP equation (48) for 60 observations**

| Fully Modified OLS estimates (MC stand. dev. below) | |
|---|---|
| Bartlett kernel | $\hat{\beta} = \begin{pmatrix} 0.95 & 1.01 & 0.96 \\ (0.12) & (0.13) & (0.12) \end{pmatrix}$ |
| Parzen kernel | $\hat{\beta} = \begin{pmatrix} 0.95 & 1.01 & 0.96 \\ (0.12) & (0.13) & (0.12) \end{pmatrix}$ |
| Quadratic Spectral kernel | $\hat{\beta} = \begin{pmatrix} -0.95 & 1.01 & 0.96 \\ (0.12) & (0.13) & (0.12) \end{pmatrix}$ |

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